Power-law distribution in budget changes: macroscopic and microscopic modeling strategies

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Observations of the political scientists

About power-law and mechanisms behind

Why is there power-law in budget changes?

Modeling framework

Macroscopic model

Microscopic modeling
Economic growth

GDP in the USA

http://www.econdash.com/images/gdp_us_graph.gif
US real outlays, logarithmic scale

Jones et al, 2007: A General Empirical Law of Public Budgets: A Comparative Analysis. Figure 2b.
US real outlays, percentage change

Jones et al, 2007: A General Empirical Law of Public Budgets: A Comparative Analysis. Figure 2a.
US total outlays, histogram

- leptokurtic
- asymmetric
- bounded
- peak close to zero

Jones et al, 2007: A General Empirical Law of Public Budgets: A Comparative Analysis. Figure 3a.
Kurtosis, platy- meso- leptokurtic

http://upload.wikimedia.org/wikipedia/commons/e/e6/Standard_symmetric_pdf.png
Kurtosis, platy- meso- leptocurtic

Plot of several symmetric unimodal probability densities with unit variance

From highest to lowest peak:

- red, kurtosis 3, Laplace (D)ouble exponential distribution
- orange, kurtosis 2, hyperbolic (S)ecant distribution
- green, kurtosis 1.2, (L)ogistic distribution
- black, kurtosis 0, (N)ormal distribution
- cyan, kurtosis $-0.593762\ldots$, raised (C)osine distribution
- blue, kurtosis $-1$, (W)igner semicircle distribution
- magenta, kurtosis $-1.2$, (U)niform distribution

http://upload.wikimedia.org/wikipedia/commons/e/e6/Standard_symmetric_pdf.png
Real US GDP quarterly change

- slightly leptokurtic
- slightly asymmetric
- peak near 0.15
  (US growth 2.20)

What is the source of kurtosis?

- **GDP**: More or less stable growth
- **Budgets**: Punctuated equilibrium, “bursts of frenetic activity” (Jones)
Identified mechanisms: friction

- Limited attention of political institutions
- Friction by collective decision rules, resistance to change
- Error accumulation or strong signals
- Prioritized preferences, re-prioritize
- Urgency causes collective attention
- “The contagion of urgency overcomes the friction of order” (Jones)
US total outlays, log-log plot

Why linear?
Our goal is to explain.

Jones et al, 2007: A General Empirical Law of Public Budgets: A Comparative Analysis. Figure 3b.
Pareto distribution

- Power function: \( y = a \cdot x^k \)
- Probability density distribution (PDF) is power function
- Linear in log-log plot:
  \[
  y = a \cdot x^k \\
  \ln y = \ln a + k \cdot \ln x \\
  Y = A + k \cdot X
  \]

Exponential growth with exp kill

- $y = a \cdot x^k$, $k \sim \exp(b)$
- Many plates with many bulbs
- Power-law distribution
Reciprocal of quantities

- y has a distribution that passes through zero
- \( x = 1 / y \)
- x has a distribution with a power law tail
- We haven't found any microscopic or macroscopic analogy in political decisions.
Random walks

- 1D random walk
- first return time

Newman, 2006, Figure 9
The Yule process

- Speciation: splitting of one species into two
- Once every $m$ speciation events sufficiently different to be considered a new genus (Newman)
- Number of species in a genus: power law
- Gibrat principle: “rich get richer”

http://www.mun.ca/biology/scarr/Stanley_Speciation_A_&_B.gif
Criticality

- $p=0.3, p=0.5927..., p=0.9$
- clusters – spanning cluster
- critical point, percolation
- power-law at the critical point
- budget power-law, because of power-law

Newman, 2006, Figures 11 and 12
Self-organized criticality

- Forest fires
- Sandpile, avalanche
- Earthquakes

Newman, 2006, Figure 13.
http://www.geo.lsa.umich.edu/~ruff/Geo105.W97/SOC/SOCeq.html
Highly optimized tolerance

- Forest fire, but trees deliberately planted to optimize the amount of lumber, random fires
- Approximate power-law
W. Reed proved that geometric Brownian motion leads to double Pareto distribution

\[ dX = \mu X \, dt + \sigma X \, dw \]

"simple, plausible explanation which can explain many examples in economics ... and other areas."
Classification of mechanisms

- Continuous process – many independent processes
- Macroscopic - microscopic mechanisms
Model framework

- Political system gets inputs, signal $S$
- Numerical budget decision is its response, $R$
- Roaming in the S-R space, stochastic
  \[ R_t = r(S_t, S_{t-1}, S_{t-2}, \ldots R_{t-1}, R_{t-2}, \ldots, e_r) \]
  \[ S_t = s(S_{t-1}, S_{t-2}, \ldots R_{t-1}, R_{t-2}, \ldots, e_s) \]
- A very simple model may give explanation
  \[ R_t = f(S_t) \]
  \[ S_t = s(e_s) \]
Combination of exponentials budget model

- \[ R = a \cdot x^S, \quad S \sim \exp(b), \quad 0 < x < 1 \]

- Direct explanation: "He gives twice who gives quickly" 
"The contagion of urgency overcomes the friction of order"

- General explanation, allows for several microscopic mechanisms
Future microscopic model

- Phenomenology came first
- Waiting times in political decisions
- Data analysis of various budget components, proposition re-entry into parliament, timings of hearings
- Various microscopic power-law mechanisms may be behind the macroscopic double exp.
Other directions to consider

- Lognormal distribution
- Approximate power-law distributions
- Highly optimized tolerance
Summary

- Studied budget changes
- Studied power-law mechanisms
- Model frame
- General, phenomenological, macroscopic model
- Towards microscopic modeling